1. Which of these are propositions? What are the truth values of those that are propositions?
   - a) Do not pass go.
   - b) What time is it?
   - c) There are seven days in a week.
   - d) $4 + x = 5.$
   - e) The moon is made of green cheese.
   - f) $2^n \geq 100.$

2. Let *p*, *q*, and *r* be the propositions
   - *P*: You get an A on the final exam.
   - *q*: You do every exercise in this book.
   - *r*: You get A in this class.

Write the propositions using *p*, *q*, and *r* and logical connectives

d) You get an A on the final, but you don't do every exercise in this book; nevertheless, you get an A in this class.

e) Getting an A on the final and doing every exercise in this book is sufficient for getting an A in this class.

f) You will get an A in this class if and only if you either do every exercise in this book or you get an A on the final.

3. Construct a truth table for the compound proposition.

$(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$

4. Are the system specifications consistent?

If the file system is not locked, then new messages will be queued.

If the file system is not locked, then the system is functioning normally, and conversely.

If new messages are not queued, then they will be sent to the message buffer.

If the file system is not locked, then new messages will be sent to the message buffer.

New messages will not be sent to the message buffer.

5. Show that $p \leftrightarrow q$ and $(p \wedge q) \vee (\neg p \wedge \neg q)$ are equivalent.

6. Show that $(p \vee q) \wedge (\neg p \vee r) \rightarrow (q \vee r)$ is a tautology.

7. Let $P(x)$ be the statement "the word x contains the letter a." What are these truth values?

a) $P(orange)$

b) $P(lemon)$

c) $P(true)$

d) $P(false)$

8. Determine the truth value of each of these statements if the domain for all variables consists of all real numbers.

a) $\exists x(x^2 = 2)$

b) $\exists x(x^2 = -1)$

c) $\forall x(x^2 + 2 \geq 1)$

d) $\forall x(x^2 \neq x)$

9. Suppose the domain of the propositional function $P(x, y)$ consists of pairs x and y where x is 1, 2, or3 and y is 1, 2, or 3. Write out these propositions using disjunctions and conjunctions.

a) $\exists x P(x, 3)$

c) $\exists y \neg P(2, y)$

10. As mentioned in the text, the notation $\exists! x P(x)$ denotes "There exists a unique x such that $P(x)$ is true." If the domain consists of all integers, what are the truth values of these statements?

a) $\exists! x(x > 1)$

b) $\exists! x(x^2 = 1)$

c) $\exists! x(x + 3 = 2x)$

d) $\exists! x(x = x + 1)$

11. Let $P(x)$, $Q(x)$, $R(x)$, and $S(x)$ be the statements "x is a duck," "x is one of my poultry," "x is an officer," and "x is willing to waltz," respectively. Express each of these statements using quantifiers; logical connectives; and $P(x)$, $Q(x)$, $R(x)$, and $S(x)$.

a) No ducks are willing to waltz.

b) No officers ever decline to waltz.

c) All my poultry are ducks.

d) My poultry are not officers.

12. Determine the truth value of each of these statements if the domain of each variables consists of all real numbers.

h) $\exists x \exists y(x + 2y = 2 \wedge 2x + 4y = 5)$

i) $\forall x \exists y(x + y = 2 \wedge 2x - y = 1)$

j) $\forall x \forall y \exists z(z = (x + y)/2)$

13. Express the negation of the statement so that all negation symbols immediately precede predicates.

$$\exists x \exists y P(x, y) \wedge \forall x \forall y Q(x, y)$$

14. Use rules of inference to show that the hypotheses.

"If it does not rain or it is not foggy, then the sailing race will be held and the lifesaving demonstration will go on,"

"If the sailing race is held, then the trophy will be awarded,"

and "The trophy was not awarded"

imply the conclusion "It rained."

TABLE 1 Rules of Inference.

Rule of Inference	*Tautology*	*Name*
p $p \to q$ $\therefore q$	$[p \wedge (p \to q)] \to q$	Modus ponens
$\neg q$ $p \to q$ $\therefore \neg p$	$[\neg q \wedge (p \to q)] \to \neg p$	Modus tollens
$p \to q$ $q \to r$ $\therefore p \to r$	$[(p \to q) \wedge (q \to r)] \to (p \to r)$	Hypothetical syllogism
$p \vee q$ $\neg p$ $\therefore q$	$[(p \vee q) \wedge \neg p] \to q$	Disjunctive syllogism
p $\therefore p \vee q$	$p \to (p \vee q)$	Addition
$p \wedge q$ $\therefore p$	$(p \wedge q) \to p$	Simplification

15. Determine whether these are valid arguments.

a) If x is a positive real number, then x^2 is a positive real number. Therefore, if a^2 is positive, where a is a real number, then a is a positive real number.

b) If $x^2 \neq 0$, where x is a real number, then $x \neq 0$. Let a be a real number with $a^2 \neq 0$; then $a \neq 0$.

16. Prove that if $m+n$ and $n+p$ are even integers, where m, n, and p are integers, then $m+p$ is even. What kind of proof did you use?

17. Prove that if n is a perfect square, then $n+2$ is not a perfect square.

18. Prove that if x is rational and $x \neq 0$, then $1/x$ is rational.

19. Prove that if m and n are integers and mn is even, then m is even or n is even.

20. Prove that if n is an integer and $3n+2$ is even, then n is even.